ONLINE APPENDIX FOR "TESTING THE QUANTAL RESPONSE HYPOTHESIS"

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A. CYCLIC MONOTONICITY AND UTILITY MAXIMIZATION

In this section we show that the CM inequalities are equivalent to players' utility maximization. In order to establish this result we exploit the fact that the set of QRE can be seen as the set of NE of a specially perturbed game. It can be shown[29] that the set of QRE corresponds to the set of NE of a game where players' payoffs are given by

$$\mathcal{G}^i(\boldsymbol{\pi}_i;\boldsymbol{u}_i) := \langle \boldsymbol{\pi}_i, \boldsymbol{u}_i \rangle - \tilde{\varphi}^i(\boldsymbol{\pi}_i), \quad \forall i \in N, \tag{14}$$

with $\tilde{\varphi}^i(\boldsymbol{\pi}_i)$ corresponding to the Fenchel-Legendre conjugate (hereafter convex conjugate) of the function $\varphi^i(\boldsymbol{u}_i)$ defined in (2).[30] By fundamental properties of the conjugacy relationship (cf. (Rockafellar, 1970, pp. 103–104)), we also have

$$\varphi^i(\boldsymbol{u}_i) = \sup_{\boldsymbol{\pi_i}} \mathcal{G}^i(\boldsymbol{\pi}_i;\boldsymbol{u}_i) = \sup_{\boldsymbol{\pi_i}} \left[\langle \boldsymbol{\pi}_i, \boldsymbol{u}_i \rangle - \tilde{\varphi}^i(\boldsymbol{\pi}_i)\right], \quad \forall i \in N, \tag{15}$$

hence $\forall i \in N$ and $\forall \boldsymbol{\pi}_i \in \Delta(S_i)$, Fenchel's inequality holds:

$$\varphi^i(\boldsymbol{u}_i) \geq \langle \boldsymbol{\pi}_i, \boldsymbol{u}_i \rangle - \tilde{\varphi}^i(\boldsymbol{\pi}_i)$$

with equality holding at $\boldsymbol{\pi}_i^* = \mathrm{argsup}_{\boldsymbol{\pi}_i} \mathcal{G}^i(\boldsymbol{\pi}_i;\boldsymbol{u}_i)$.

Proof of Proposition 1: Utility maximization in each perturbed game implies CM: Consider a cycle of of unperturbed games of length $\mathcal{L}-1$ with $[\boldsymbol{u}_i]^m$ and $[\boldsymbol{\pi}_i^*]^m$ denoting the expected payoffs and equilibrium probabilities in an unperturbed game indexed m in the cycle, respectively. By utility maximization in the corresponding perturbed game, it is easy to see that for each player i the following inequalities must hold:

$$\langle [\boldsymbol{u}_i]^{m+1}, [\boldsymbol{\pi}_i^*]^m \rangle - \tilde{\varphi}^i([\boldsymbol{\pi}_i^*]^m) \leq \langle [\boldsymbol{u}_i]^{m+1}, [\boldsymbol{\pi}_i^*]^{m+1} \rangle - \tilde{\varphi}^i([\boldsymbol{\pi}_i^*]^{m+1}), \quad \forall m. \tag{16}$$

Rewriting as

$$\langle [\boldsymbol{u}_i]^{m+1} - [\boldsymbol{u}_i]^m, [\boldsymbol{\pi}_i^*]^m \rangle \leq \langle [\boldsymbol{u}_i]^{m+1}, [\boldsymbol{\pi}_i^*]^{m+1} \rangle - \langle [\boldsymbol{u}_i]^m, [\boldsymbol{\pi}_i^*]^m \rangle + \tilde{\varphi}^i([\boldsymbol{\pi}_i^*]^m) - \tilde{\varphi}^i([\boldsymbol{\pi}_i^*]^{m+1}),$$

and adding up over the cycle, we get the CM inequalities (4).

CM implies utility maximization: Suppose that CM holds and let $[\boldsymbol{u}_i]^m$ denote expected utility in an unperturbed game m. By (Rockafellar, 1970, Theorems 24.8 and 24.9) we know that there exists a closed convex function ζ^i such that $[\boldsymbol{\pi}_i^*]^m = \partial\zeta^i([\boldsymbol{u}_i]^m)$. By (Rockafellar, 1970, Theorem 23.5) since $[\boldsymbol{\pi}_i^*]^m$ is in the subdifferential of ζ^i evaluated at $[\boldsymbol{u}_i]^m$, the Fenchel's inequality is satisfied as an equation at $([\boldsymbol{\pi}_i^*]^m, [\boldsymbol{u}_i]^m)$, i.e.,

$$\langle [\boldsymbol{\pi}_i^*]^m, [\boldsymbol{u}_i]^m \rangle - \tilde{\zeta}^i([\boldsymbol{\pi}_i^*]^m) = \zeta^i([\boldsymbol{u}_i]^m).$$

Condition b^*) of (Rockafellar, 1970, Theorem 23.5) implies that $[\boldsymbol{\pi}_i^*]^m \in \arg\sup_{\boldsymbol{\pi}_i}\{\langle \boldsymbol{\pi}_i, [\boldsymbol{u}_i]^m \rangle - \tilde{\zeta}^i(\boldsymbol{\pi}_i)\}$, i.e.,

$$\zeta^i([\boldsymbol{u}_i]^m) = \sup_{\boldsymbol{\pi_i}} \left[\langle \boldsymbol{\pi}_i, [\boldsymbol{u}_i]^m \rangle - \tilde{\zeta}^i(\boldsymbol{\pi}_i)\right]$$

Comparing this with Eq. (15) above, by (Rockafellar, 1970, Theorem 24.9), we conclude that, up to an additive constant, the function ζ (resp. $\tilde{\zeta}$) is identical to φ (resp. $\tilde{\varphi}$), therefore $[\boldsymbol{\pi}_i^*]^m \in \arg\sup_{\boldsymbol{\pi}_i}\{\mathcal{G}^i(\boldsymbol{\pi}_i;[\boldsymbol{u}_i]^m)\}$.

[29]In particular, Cominetti et al. (2010, Prop. 3) establish a way of representing a Logit QRE (and more generally, a structural QRE) as a Nash equilibrium of the game with payoffs (14). See also Hofbauer and Sandholm (2002, Thm. 2.1) for a related result in the single-agent case.

[30]Formally, for a convex function f the Fenchel-Legendre conjugate is defined by $\tilde{f}(\boldsymbol{y}) = \sup_{\boldsymbol{x}}\{\langle \boldsymbol{y}, \boldsymbol{x} \rangle - f(\boldsymbol{x})\}$.

That is, CM implies utility maximization in the perturbed game for each m. □

Intuitively, the set of inequalities (16) can be seen as set of incentive compatibility constraints across the series of games that only differ in the payoffs. This means that our CM conditions capture players' optimization behavior with respect to changes in expected payoffs across such games.

B. PROOF OF PROPOSITION 2

Suppose there are two games that differ only in the payoffs. For $M = 2$, the cyclic monotonicity condition (5) reduces to

$$\sum_{j=1}^{J_i}(u_{ij}^1 - u_{ij}^0)\pi_{ij}^0 + \sum_{j=1}^{J_i}(u_{ij}^0 - u_{ij}^1)\pi_{ij}^1 \leq 0,$$

or, equivalently,

$$\sum_{j=1}^{J_i}(u_{ij}^1 - u_{ij}^0)(\pi_{ij}^0 - \pi_{ij}^1) \leq 0. \tag{17}$$

Note that cyclic monotonicity inequality in Eq. (17) is distinct from HHK's cumulative rank condition (8). Suppose that the right-hand side of (7) is non-negative. Then HHK condition (8) implies CM. To see this, notice that for non-negative utilities differences in (7)

$$(u_{i1}^1 - u_{i1}^0)(\pi_{i1}^0 - \pi_{i1}^1) \leq 0$$

by HHK condition for $k = 1$. Then

$$\begin{aligned}(u_{i2}^1 - u_{i2}^0)(\pi_{i2}^0 - \pi_{i2}^1) + (u_{i1}^1 - u_{i1}^0)(\pi_{i1}^0 - \pi_{i1}^1) \leq \\ (u_{i2}^1 - u_{i2}^0)(\pi_{i2}^0 - \pi_{i2}^1) + (u_{i2}^1 - u_{i2}^0)(\pi_{i1}^0 - \pi_{i1}^1) = \\ (u_{i2}^1 - u_{i2}^0)((\pi_{i1}^0 + \pi_{i2}^0) - (\pi_{i1}^1 + \pi_{i2}^1)) \leq 0\end{aligned}$$

where the last inequality follows from HHK condition for $k = 2$ and $u_{i2}^1 - u_{i2}^0 \geq 0$. Repeating the same procedure for $k = 3, \ldots, J_i$, we obtain the CM condition (17) for $M = 2$.

Conversely, suppose that (17) holds. For the case of two games, (17) holding for all players is necessary and sufficient to generate QRE-consistent choices. All premises are satisfied for HHK's Theorem 2, so condition (8) follows. One can also show it directly. Clearly, given (17), we can always re-label strategy indices so that (7) holds. Let $k = 1$ and by way of contradiction, suppose that (8) is violated, i.e. $\pi_{i1}^1 - \pi_{i1}^0 < 0$. Since (17) holds, the probabilities in both games are generated by a QRE. Due to indexing in (7),

$$u_{i1}^1 - u_{ij}^1 \geq u_{i1}^0 - u_{ij}^0$$

for all $j > 1$. But then by definition of QRE in (1), $\pi_{i1}^1 \geq \pi_{i1}^0$. Contradiction, so (8) holds for $k = 1$. By induction on the strategy index, one can show that (8) holds for all $k \in \{1, \ldots, J_i\}$. □

C. UNIQUENESS OF REGULAR QRE IN EXPERIMENTAL JOKER GAMES

In this section we show formally that any regular QRE in Games 1–4 from Table 1 is unique. We start by recalling the necessary definitions.

A quantal response function $\pi_i : \mathbb{R}^{J_i} \to \Delta(S_i)$ is *regular*, if it satisfies Interiority, Continuity, Responsiveness, and Rank-order[31] axioms (Goeree et al., 2005, p.355). Interiority, Continuity, and Responsiveness are satisfied automatically under the *structural approach* to quantal response[32] that we pursue in this paper as long as the shock distributions have full support. Importantly, for some shock distributions this approach may fail to satisfy the Rank-order Axiom, i.e. the intuitive property of QRE saying that actions with higher

[31] This property is called Monotonicity in Goeree et al. (2005).

[32] In this approach, the quantal response functions are derived from the primitives of the model with additive payoff shocks, as described in Section 2.

expected payoffs are played with higher probability than actions with lower expected payoffs. For the sake of convenience, we repeat the axiom here.

A quantal response function $\pi_i : \mathbb{R}^{J_i} \to \Delta(S_i)$ satisfies *Rank-order Axiom* if for all $i \in N, j, k \in \{1, \ldots, J_i\}$ $u_{ij}(\mathbf{p}) > u_{ik}(\mathbf{p}) \Rightarrow \pi_{ij}(\mathbf{p}) > \pi_{ik}(\mathbf{p})$.

Notice that the Rank-order Axiom involves comparisons of expected payoffs from choosing different pure strategies *within* a fixed game. As briefly discussed in Section 4.2, consistency of the data with the Rank-order Axiom can be tested: the Axiom is equivalent to the following inequality for each player i and pair of i's strategies $j, k \in \{1, \ldots, J_i\}$:

$$(u_{ij}(\mathbf{p}) - u_{ik}(\mathbf{p}))(\pi_{ij}(\mathbf{p}) - \pi_{ik}(\mathbf{p})) \geq 0 \tag{18}$$

Thus a modified test for consistency with a *regular* QRE involves two stages: first, check if the data are consistent with a structural QRE using the cyclic monotonicity inequalities (which compare choices across games) as described in Section 5, and second, if the test does not reject the null hypothesis of consistency, check if the Rank-order Axiom (which compares choices within a game) holds by estimating (18) for each game.[33]

Alternatively, the Rank-order Axiom can be imposed from the outset by making an extra assumption about the shock distributions. In particular, Goeree et al. (2005, Proposition 5) shows that under the additional assumption of exchangeability, the quantal response functions derived under the structural approach are regular.

Notice that Assumption 1 (Invariance) is not required for the test of the Rank-order Axiom. In theory, we may have cases where the data can be rationalized by a structural QRE that fails Rank-order, by a structural QRE that satisfies it (i.e. by a regular QRE), or by quantal response functions that satisfy Rank-order in each game but violate the assumption of fixed shock distributions across games. In the latter case, checking consistency with other boundedly rational models (e.g., Level-k or Cognitive Hierarchy) becomes a natural follow-up step.

We can now turn to the uniqueness of the regular QRE in our test games.

Proposition 3. *For each player $i \in N \equiv \{Row, Col\}$ fix a regular quantal response function $\pi_i : \mathbb{R}^{J_i} \to \Delta(S_i)$ and let $Q \equiv (\pi_i)_{i \in N}$. In each of Games 1–4 from Table 1 there is a unique quantal response equilibrium (σ^R, σ^C) with respect to Q. Moreover, in Game 1, the unique quantal response equilibrium is $\sigma^R = \sigma^C = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. In Game 2, $\sigma_1^R = \sigma_2^R \in \left(0, \frac{1}{3}\right)$ and $\sigma_1^C = \sigma_2^C \in \left(\frac{1}{3}, \frac{9}{22}\right]$. In Game 3, $\sigma_1^R = \sigma_2^R \in \left(\frac{1}{3}, 1\right)$ and $\sigma_1^C = \sigma_2^C \in \left[\frac{4}{15}, \frac{1}{3}\right)$. In Game 4, $\sigma_2^R = \sigma_J^R \in \left(0, \frac{1}{3}\right)$ and $\sigma_1^C = \sigma_J^C \in \left(\frac{1}{3}, \frac{2}{5}\right]$.*

Notice that the equilibrium probability constraints in Proposition 3 hold in any regular QRE, not only logit QRE. For the logit QRE they hold for any scale parameter $\lambda \in [0, \infty)$.

Proof. In order to prove uniqueness and bounds on QRE probabilities we will be mainly using Rank-order and Responsiveness properties of a regular QRE.

Suppose Row plays $\sigma^R = (\sigma_1^R, \sigma_2^R, \sigma_J^R)$, Col plays $\sigma^C = (\sigma_1^C, \sigma_2^C, \sigma_J^C)$, and (σ^R, σ^C) is a regular QRE.[34] Expected utility of Col from choosing each of her three pure strategies in any of Games 1–4 (see payoffs in Table 1) is

$$\begin{aligned} u_{C1}(\sigma^R) &= 30 - 20\sigma_2^R \\ u_{C2}(\sigma^R) &= 30 - 20\sigma_1^R \\ u_{CJ}(\sigma^R) &= 10 + 20\sigma_1^R + 20\sigma_2^R \end{aligned}$$

Consider Game 1. Expected utility of Row in this game from choosing each of her three pure strategies is

$$\begin{aligned} u_{R1}(\sigma^C) &= 10 + 20\sigma_2^C \\ u_{R2}(\sigma^C) &= 10 + 20\sigma_1^C \\ u_{RJ}(\sigma^C) &= 30 - 20\sigma_1^C - 20\sigma_2^C \end{aligned}$$

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[33] The test procedure is similar to the one in Section 5, with an appropriately modified Jacobian matrix.

[34] Obviously, $\sigma_J^R = 1 - \sigma_1^R - \sigma_2^R$ and $\sigma_J^C = 1 - \sigma_1^C - \sigma_2^C$.

Consider Row's equilibrium strategy. There are two possibilities: 1) $\sigma_1^R > \sigma_2^R$. Then Rank-order applied to Col implies $\sigma_2^C < \sigma_1^C$. Now Rank-order applied to Row implies $\sigma_1^R < \sigma_2^R$. Contradiction. 2) $\sigma_1^R < \sigma_2^R$. Then Rank-order applied to Col implies $\sigma_2^C > \sigma_1^C$. Now Rank-order applied to Row implies $\sigma_1^R > \sigma_2^R$. Contradiction. Therefore, in *any* regular QRE in Game 1, $\sigma_1^R = \sigma_2^R$, and consequently, $\sigma_1^C = \sigma_2^C$. Suppose $\sigma_1^R > \frac{1}{3}$. Then Rank-order applied to Col implies $\sigma_J^C > \sigma_1^C$, and since $\sigma_J^C = 1 - 2\sigma_1^C$, we have $\frac{1}{3} > \sigma_1^C$. Then Rank-order applied to Row implies $\sigma_1^R < \sigma_J^R$, and so $\sigma_1^R < \frac{1}{3}$. Contradiction. Suppose $\sigma_1^R < \frac{1}{3}$. Then Rank-order applied to Col implies $\sigma_J^C < \sigma_1^C$, and so $\frac{1}{3} < \sigma_1^C$. Then Rank-order applied to Row implies $\sigma_1^R > \sigma_J^R$, and so $\sigma_1^R > \frac{1}{3}$. Contradiction. Therefore $\sigma_1^R = \frac{1}{3}$, and hence $\sigma^R = \sigma^C = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ in *any* regular QRE, so the equilibrium is unique.

Consider Game 2. Row's expected utility in this game from choosing each of her three pure strategies is

$$\begin{aligned} u_{R1}(\sigma^C) &= 10 + 20\sigma_2^C \\ u_{R2}(\sigma^C) &= 10 + 20\sigma_1^C \\ u_{RJ}(\sigma^C) &= 55 - 45\sigma_1^C - 45\sigma_2^C \end{aligned}$$

The previous analysis immediately implies that in *any* regular QRE, $\sigma_1^R = \sigma_2^R$, and $\sigma_1^C = \sigma_2^C$. Suppose $\sigma_1^R \geq \frac{1}{3}$. Then Rank-order applied to Col implies $\sigma_1^C \leq \sigma_J^C$, and since $\sigma_J^C = 1 - 2\sigma_1^C$, we have $\sigma_1^C \leq \frac{1}{3}$. Then Rank-order applied to Row implies $\sigma_1^R < \sigma_J^R$, so $\sigma_1^R < \frac{1}{3}$. Contradiction. Therefore, $\sigma_1^R < \frac{1}{3}$. Then Rank-order applied to Col implies $\sigma_1^C > \sigma_J^C$, and since $\sigma_J^C = 1 - 2\sigma_1^C$, we have $\sigma_1^C > \frac{1}{3}$. If we also had $\sigma_1^C > \frac{9}{22}$, then Rank-order applied to Row would imply $\sigma_1^R > \sigma_J^R$, hence $\sigma_1^R > \frac{1}{3}$, contradiction. Thus in any regular QRE, $\frac{1}{3} < \sigma_1^C \leq \frac{9}{22}$ and $\sigma_1^R < \frac{1}{3}$. It remains to prove that σ_1^R and σ_1^C are uniquely defined. Applying Responsiveness to Col implies that σ_1^C is strictly increasing in U_{C1}, and therefore is strictly decreasing in σ_1^R. Using the same argument for Row, σ_1^R is strictly increasing in σ_1^C. Therefore any regular QRE in Game 2 is unique.

Consider Game 3. Row's expected utility in this game from choosing each of her three pure strategies is

$$\begin{aligned} u_{R1}(\sigma^C) &= 10 + 15\sigma_1^C + 20\sigma_2^C \\ u_{R2}(\sigma^C) &= 10 + 20\sigma_1^C + 15\sigma_2^C \\ u_{RJ}(\sigma^C) &= 30 - 20\sigma_1^C - 20\sigma_2^C \end{aligned}$$

As before, it is easy to show that in *any* regular QRE, $\sigma_1^R = \sigma_2^R$, and therefore $\sigma_1^C = \sigma_2^C$. Applying Responsiveness to Col implies that σ_1^C is strictly increasing in U_{C1}, and therefore is strictly decreasing in σ_1^R. Using the same argument for Row, σ_1^R is strictly increasing in σ_1^C. Therefore any regular QRE in Game 3 is unique. To prove the bounds on QRE probabilities, suppose $\sigma_1^R \leq \frac{1}{3}$. Then Rank-order applied to Col implies $\sigma_1^C \geq \sigma_J^C$, and since $\sigma_J^C = 1 - 2\sigma_1^C$, we have $\sigma_1^C \geq \frac{1}{3}$. Then Rank-order applied to Row implies $\sigma_1^R > \sigma_J^R$, so $\sigma_1^R > \frac{1}{3}$. Contradiction. Therefore, $\sigma_1^R > \frac{1}{3}$. Then Rank-order applied to Col implies $\sigma_1^C < \sigma_J^C$, and since $\sigma_J^C = 1 - 2\sigma_1^C$, we have $\sigma_1^C < \frac{1}{3}$. If we also had $\sigma_1^C < \frac{4}{15}$, then Rank-order applied to Row would imply $\sigma_1^R < \sigma_J^R$, hence $\sigma_1^R < \frac{1}{3}$, contradiction. Thus in any regular QRE, $\frac{4}{15} \leq \sigma_1^C < \frac{1}{3}$ and $\sigma_1^R > \frac{1}{3}$.

Finally, consider Game 4. We will now write $\sigma_2^C = 1 - \sigma_1^C - \sigma_J^C$, then the expected utility of Row in this game from choosing each of her three pure strategies is

$$\begin{aligned} u_{R1}(\sigma^C) &= 30 - 10\sigma_1^C - 20\sigma_J^C \\ u_{R2}(\sigma^C) &= 10 + 20\sigma_1^C \\ u_{RJ}(\sigma^C) &= 10 + 20\sigma_J^C \end{aligned}$$

Consider Col's equilibrium strategy. There are two possibilities: 1) $\sigma_1^C > \sigma_J^C$. Then Rank-order applied to Row implies $\sigma_2^R > \sigma_J^R$, hence $\sigma_1^R + 2\sigma_2^R > 1$. Then Rank-order applied to Col implies $\sigma_1^C < \sigma_J^C$. Contradiction. 2) $\sigma_1^C < \sigma_J^C$. Then Rank-order applied to Row implies $\sigma_2^R < \sigma_J^R$, hence $\sigma_1^R + 2\sigma_2^R < 1$. Then Rank-order applied to Col implies $\sigma_1^C > \sigma_J^C$. Contradiction. Therefore, in *any* regular QRE in Game 4, $\sigma_1^C = \sigma_J^C$, and consequently, $\sigma_2^R = \sigma_J^R$ (or, equivalently, $\sigma_1^R + 2\sigma_2^R = 1$). Applying Responsiveness to Row, σ_J^R is strictly increasing in U_{RJ}, and therefore is strictly increasing in $\sigma_J^C \equiv \sigma_1^C$. Using the same argument for Col, σ_1^C is strictly decreasing in $\sigma_2^R \equiv \sigma_J^R$. Therefore any regular QRE in Game 4 is unique. To prove the bounds on QRE probabilities, suppose $\sigma_2^R \geq \sigma_1^R$. Then by Rank-order applied to Col, $\sigma_1^C \leq \sigma_2^C$ and so

$\sigma_1^C \leq \frac{1}{3}$. But then Rank-order applied to Row implies $\sigma_2^R < \sigma_1^R$. Contradiction. Hence $\sigma_2^R < \sigma_1^R$, and so $\sigma_2^R < \frac{1}{3}$. Then Rank-order applied to Col implies $\sigma_1^C > \sigma_2^C$, and so $\sigma_1^C > \frac{1}{3}$. If $\sigma_1^C > \frac{2}{5}$, then by Rank-order $\sigma_1^R < \sigma_2^R$. Contradiction. Therefore $\frac{1}{3} < \sigma_1^C \leq \frac{2}{5}$ and $\sigma_2^R < \frac{1}{3}$. □

D. Additional Details for Computing the Test Statistic

As defined in the main text, the P-dimensional vector $\boldsymbol{\nu}$ contains the value of the CM inequalities evaluated at the choice frequencies observed in the experimental data. Specifically, the ℓ-th component of $\boldsymbol{\nu}$, corresponding to a given cycle $G_0, \ldots, G_{\mathcal{L}}$ of games is given by

$$\begin{aligned}
\nu_\ell = \sum_{m=G_0}^{G_{\mathcal{L}}} & \pi_{i1}^m \left[\pi_{k1}^{m+1} u_i^{m+1}(s_{i1}, s_{k1}) - \pi_{k1}^m u_i^m(s_{i1}, s_{k1}) + \pi_{k2}^{m+1} u_i^{m+1}(s_{i1}, s_{k2}) - \pi_{k2}^m u_i^m(s_{i1}, s_{k2})\right. \\
& + \left.(1 - \pi_{k1}^{m+1} - \pi_{k2}^{m+1}) u_i^{m+1}(s_{i1}, s_{kJ}) - (1 - \pi_{k1}^m - \pi_{k2}^m) u_i^m(s_{i1}, s_{kJ})\right] \\
& + \pi_{i2}^m \left[\pi_{k1}^{m+1} u_i^{m+1}(s_{i2}, s_{k1}) - \pi_{k1}^m u_i^m(s_{i2}, s_{k1}) + \pi_{k2}^{m+1} u_i^{m+1}(s_{i2}, s_{k2}) - \pi_{k2}^m u_i^m(s_{i2}, s_{k2})\right. \\
& + \left.(1 - \pi_{k1}^{m+1} - \pi_{k2}^{m+1}) u_i^{m+1}(s_{i2}, s_{kJ}) - (1 - \pi_{k1}^m - \pi_{k2}^m) u_i^m(s_{i2}, s_{kJ})\right] \\
& + (1 - \pi_{i1}^m - \pi_{i2}^m) \left[\pi_{k1}^{m+1} u_i^{m+1}(s_{iJ}, s_{k1}) - \pi_{k1}^m u_i^m(s_{iJ}, s_{k1})\right. \\
& + \pi_{k2}^{m+1} u_i^{m+1}(s_{iJ}, s_{k2}) - \pi_{k2}^m u_i^m(s_{iJ}, s_{k2}) \\
& + \left.(1 - \pi_{k1}^{m+1} - \pi_{k2}^{m+1}) u_i^{m+1}(s_{iJ}, s_{kJ}) - (1 - \pi_{k1}^m - \pi_{k2}^m) u_i^m(s_{iJ}, s_{kJ})\right]
\end{aligned}$$

where we use i to denote the Row player, k to denote the Column player, and ℓ changes from 1 to 20. For the Column player and $\ell \in [21, 40]$ the analogous expression is as follows:

$$\begin{aligned}
\nu_\ell = \sum_{m=G_0}^{G_{\mathcal{L}}} & \pi_{k1}^m \left[\pi_{i1}^{m+1} u_k^{m+1}(s_{i1}, s_{k1}) - \pi_{i1}^m u_k^m(s_{i1}, s_{k1}) + \pi_{i2}^{m+1} u_k^{m+1}(s_{i2}, s_{k1}) - \pi_{i2}^m u_k^m(s_{i2}, s_{k1})\right. \\
& + \left.(1 - \pi_{i1}^{m+1} - \pi_{i2}^{m+1}) u_k^{m+1}(s_{iJ}, s_{k1}) - (1 - \pi_{i1}^m - \pi_{i2}^m) u_k^m(s_{iJ}, s_{k1})\right] \\
& + \pi_{k2}^m \left[\pi_{i1}^{m+1} u_k^{m+1}(s_{i1}, s_{k2}) - \pi_{i1}^m u_k^m(s_{i1}, s_{k2}) + \pi_{i2}^{m+1} u_k^{m+1}(s_{i2}, s_{k2}) - \pi_{i2}^m u_k^m(s_{i2}, s_{k2})\right. \\
& + \left.(1 - \pi_{i1}^{m+1} - \pi_{i2}^{m+1}) u_k^{m+1}(s_{iJ}, s_{k2}) - (1 - \pi_{i1}^m - \pi_{i2}^m) u_k^m(s_{iJ}, s_{k2})\right] \\
& + (1 - \pi_{k1}^m - \pi_{k2}^m) \left[\pi_{i1}^{m+1} u_k^{m+1}(s_{i1}, s_{kJ}) - \pi_{i1}^m u_k^m(s_{i1}, s_{kJ})\right. \\
& + \pi_{i2}^{m+1} u_k^{m+1}(s_{i2}, s_{kJ}) - \pi_{i2}^m u_k^m(s_{i2}, s_{kJ}) \\
& + \left.(1 - \pi_{i1}^{m+1} - \pi_{i2}^{m+1}) u_k^{m+1}(s_{iJ}, s_{kJ}) - (1 - \pi_{i1}^m - \pi_{i2}^m) u_k^m(s_{iJ}, s_{kJ})\right]
\end{aligned}$$

We differentiate the above expressions with respect to π^m to obtain a $P \times 16$ estimate of the Jacobian $\hat{J} = \frac{\partial}{\partial \boldsymbol{\pi}} \boldsymbol{\mu}(\hat{\boldsymbol{\pi}})$ in order to compute an estimate of the variance-covariance matrix $\hat{\Sigma}_{[P \times P]} = \hat{J} \hat{V} \hat{J}'$ by the Delta method. For the case of four games, the partial derivatives form the 40×16 matrix $\hat{J}$. The first 20 rows correspond to the differentiated LHS of the cycles for the Row player, and the last 20 rows correspond to the differentiated LHS of the cycles for the Column player. The first 8 columns correspond to the derivatives with respect to π_{i1}^m, π_{i2}^m, and the last 8 columns correspond to the derivatives with respect to π_{k1}^m, π_{k2}^m, $m \in \{1, \ldots, 4\}$.[35]

Let $S_0^m \equiv \{\ell \in \{1, \ldots, 40\} | m \notin C_\ell\}$ be the set of cycle indices such that corresponding cycles (in the order given in (13)) do not include game m. E.g., for $m = 1$, $S_0^m = \{4, 5, 6, 13, 14, 24, 25, 26, 33, 34\}$. Let $S_i^m \equiv \{\ell \in \{1, \ldots, 20\} | \ell \notin S_0^m\}$ be a subset of cycle indices that include game m and pertain to the Row player, and let $S_k^m \equiv \{\ell \in \{21, \ldots, 40\} | \ell \notin S_0^m\}$ be a subset of cycle indices that include game m and pertain to the Column player. Finally, for a cycle of length $\mathcal{L}$, denote $\ominus \equiv -\mod \mathcal{L}$ subtraction modulus $\mathcal{L}$.

We can now express the derivatives with respect to π_{i1}^m and π_{i2}^m, $m \in \{1, \ldots, 4\}$, in the following general

[35] Clearly, the probability to choose Joker can be expressed via the probabilities to choose 1 and 2, using the total probability constraint.

form. The partial derivatives wrt π_{i1}^m are

$$\frac{\partial \nu_\ell}{\partial \pi_{i1}^m} = 0 \qquad \text{for } \ell \in S_0^m$$

$$\begin{aligned}\frac{\partial \nu_\ell}{\partial \pi_{i1}^m} &= \pi_{k1}^{m+1}u_i^{m+1}(s_{i1}, s_{k1}) - \pi_{k1}^m u_i^m(s_{i1}, s_{k1}) + \pi_{k2}^{m+1}u_i^{m+1}(s_{i1}, s_{k2})\\ &- \pi_{k2}^m u_i^m(s_{i1}, s_{k2}) + (1 - \pi_{k1}^{m+1} - \pi_{k2}^{m+1})u_i^{m+1}(s_{i1}, s_{kJ}) - (1 - \pi_{k1}^m - \pi_{k2}^m)u_i^m(s_{i1}, s_{kJ})\\ &- \big[\pi_{k1}^{m+1}u_i^{m+1}(s_{iJ}, s_{k1}) - \pi_{k1}^m u_i^m(s_{iJ}, s_{k1}) + \pi_{k2}^{m+1}u_i^{m+1}(s_{iJ}, s_{k2}) - \pi_{k2}^m u_i^m(s_{iJ}, s_{k2})\\ &+ (1 - \pi_{k1}^{m+1} - \pi_{k2}^{m+1})u_i^{m+1}(s_{iJ}, s_{kJ}) - (1 - \pi_{k1}^m - \pi_{k2}^m)u_i^m(s_{iJ}, s_{kJ})\big] \qquad \text{for } \ell \in S_i^m\end{aligned}$$

$$\begin{aligned}\frac{\partial \nu_\ell}{\partial \pi_{i1}^m} &= \pi_{k1}^m\left[-u_k^m(s_{i1}, s_{k1}) + u_k^m(s_{iJ}, s_{k1})\right] + \pi_{k2}^m\left[-u_k^m(s_{i1}, s_{k2}) + u_k^m(s_{iJ}, s_{k2})\right]\\ &+ (1 - \pi_{k1}^m - \pi_{k2}^m)\left[-u_k^m(s_{i1}, s_{kJ}) + u_k^m(s_{iJ}, s_{kJ})\right]\\ &+ \pi_{k1}^{m\ominus1}\left[u_k^m(s_{i1}, s_{k1}) - u_k^m(s_{iJ}, s_{k1})\right] + \pi_{k2}^{m\ominus1}\left[u_k^m(s_{i1}, s_{k2}) - u_k^m(s_{iJ}, s_{k2})\right]\\ &+ (1 - \pi_{k1}^{m\ominus1} - \pi_{k2}^{m\ominus1})\left[u_k^m(s_{i1}, s_{kJ}) - u_k^m(s_{iJ}, s_{kJ})\right] \qquad \text{for } \ell \in S_k^m\end{aligned}$$

The partial derivatives wrt π_{i2}^m are

$$\frac{\partial \nu_\ell}{\partial \pi_{i2}^m} = 0 \qquad \text{for } \ell \in S_0^m$$

$$\begin{aligned}\frac{\partial \nu_\ell}{\partial \pi_{i2}^m} &= \big[\pi_{k1}^{m+1}u_i^{m+1}(s_{i2}, s_{k1}) - \pi_{k1}^m u_i^m(s_{i2}, s_{k1}) + \pi_{k2}^{m+1}u_i^{m+1}(s_{i2}, s_{k2}) - \pi_{k2}^m u_i^m(s_{i2}, s_{k2})\\ &+ (1 - \pi_{k1}^{m+1} - \pi_{k2}^{m+1})u_i^{m+1}(s_{i2}, s_{kJ}) - (1 - \pi_{k1}^m - \pi_{k2}^m)u_i^m(s_{i2}, s_{kJ})\big]\\ &- \big[\pi_{k1}^{m+1}u_i^{m+1}(s_{iJ}, s_{k1}) - \pi_{k1}^m u_i^m(s_{iJ}, s_{k1}) + \pi_{k2}^{m+1}u_i^{m+1}(s_{iJ}, s_{k2}) - \pi_{k2}^m u_i^m(s_{iJ}, s_{k2})\\ &+ (1 - \pi_{k1}^{m+1} - \pi_{k2}^{m+1})u_i^{m+1}(s_{iJ}, s_{kJ}) - (1 - \pi_{k1}^m - \pi_{k2}^m)u_i^m(s_{iJ}, s_{kJ})\big] \qquad \text{for } \ell \in S_i^m\end{aligned}$$

$$\begin{aligned}\frac{\partial \nu_\ell}{\partial \pi_{i2}^m} &= \pi_{k1}^m\left[-u_k^m(s_{i2}, s_{k1}) + u_k^m(s_{iJ}, s_{k1})\right] + \pi_{k2}^m\left[-u_k^m(s_{i2}, s_{k2}) + u_k^m(s_{iJ}, s_{k2})\right]\\ &+ (1 - \pi_{k1}^m - \pi_{k2}^m)\left[-u_k^m(s_{i2}, s_{kJ}) + u_k^m(s_{iJ}, s_{kJ})\right]\\ &+ \pi_{k1}^{m\ominus1}\left[u_k^m(s_{i2}, s_{k1}) - u_k^m(s_{iJ}, s_{k1})\right] + \pi_{k2}^{m\ominus1}\left[u_k^m(s_{i2}, s_{k2}) - u_k^m(s_{iJ}, s_{k2})\right]\\ &+ (1 - \pi_{k1}^{m\ominus1} - \pi_{k2}^{m\ominus1})\left[u_k^m(s_{i2}, s_{kJ}) - u_k^m(s_{iJ}, s_{kJ})\right] \qquad \text{for } \ell \in S_k^m\end{aligned}$$

To obtain the derivatives with respect to π_{k1}^m and π_{k2}^m, one just needs to use the corresponding partial derivatives wrt π_{i1}^m and π_{i2}^m, and exchange everywhere the subscripts i and k, so we omit the derivation. For the sake of completeness we list the cycle index subsets for each game $m \in \{1, .., 4\}$ in Table 6.

E. TESTING CYCLIC MONOTONICITY USING THE TWO-STEP APPROACH BY ROMANO, SHAIKH, AND WOLF (2014).

As an extra robustness check, we tested cyclic monotonicity conditions using another recent procedure developed by Romano et al. (2014) (henceforth referred to as RSW) for testing moment inequalities. Here, we describe the simplified version from the supplemental appendix of RSW, which applies to our setting. This is the case where we have a random vector $\boldsymbol{\mu} \in \mathbb{R}^P$ which is (asymptotically) distributed $N(\boldsymbol{\mu_0}, \Sigma)$, where $\boldsymbol{\mu_0}$ is unknown. We want to test

$$H_0: \ \boldsymbol{\mu_0} \geq \mathbf{0} \quad vs. \quad H_1: \ \boldsymbol{\mu_0} \ngeq \mathbf{0},$$

where $\mathbf{0} \in \mathbb{R}^P$. The relevant algorithm is described in section S.1.2 in the supplemental appendix of RSW.

First, for ease of comparison with RSW, we redefine $\boldsymbol{\mu} = -\boldsymbol{\mu}$, because RSW consider the complementary hypothesis

$$H_0: \ \boldsymbol{\mu_0} \leq \mathbf{0} \quad vs. \quad H_1: \ \boldsymbol{\mu_0} \nleq \mathbf{0}.$$

Letting $\hat{\Sigma}$ denote an estimate of Σ, we use the test statistic $T(\hat{\boldsymbol{\mu}}, \hat{\Sigma}) = \sum_{j=1}^P \left[\frac{\hat{\mu}_j}{\hat{\sigma}_j}\right]_+^2$, where $[x]_-$ denotes $x \cdot \mathbb{1}(x < 0)$, and $\hat{\sigma}_j$ is the square-root of the j-th diagonal entry in $\hat{\Sigma}$.

Table 6: Sets of cycle indices for each game.

m	S_0^m	S_i^m	S_k^m
1	4, 5, 6, 13, 14,	1, 2, 3, 7, 8,	21, 22, 23, 27, 28,
	24, 25, 26, 33, 34	9, 10, 11, 12, 15,	29, 30, 31, 32, 35,
		16, 17, 18, 19, 20	36, 37, 38, 39, 40
2	2, 3, 6, 10, 12,	1, 4, 5, 7, 8,	21, 24, 25, 27, 28,
	22, 23, 26, 30, 32	9, 11, 13, 14, 15,	29, 31, 33, 34, 35,
		16, 17, 18, 19, 20	36, 37, 38, 39, 40
3	1, 3, 5, 8, 11,	2, 4, 6, 7, 9,	22, 24, 26, 27, 29,
	21, 23, 25, 28, 31	10, 12, 13, 14, 15,	30, 32, 33, 34, 35,
		16, 17, 18, 19, 20	36, 37, 38, 39, 40
4	1, 2, 4, 7, 9	3, 5, 6, 8, 10,	23, 25, 26, 28, 30,
	21, 22, 24, 27, 29	11, 12, 13, 14, 15,	31, 32, 33, 34, 35,
		16, 17, 18, 19, 20	36, 37, 38, 39, 40

Notes. The cycle indices are for the Row player. To obtain the corresponding Column player cycle indices, swap the last two columns.

There are now two choice variables: α, the size of the test, which would be 0.05; and β, the size of the first-step "moment selection" procedure, $0 \leq \beta < \alpha$. β should be small – we use $\beta = 0.01$. There are three main steps (see S.1.2 in RSW for notation):

1. Compute the quantity $K^{-1}(1-\beta)$ by simulation: draw vectors $Z_s \sim N(0, \Sigma)$, for $s = 1, \ldots S$, and for each one, compute the maximal element $Z_s^* = \max(Z_{s,1}, \ldots, Z_{s,P})$. Then $K^{-1}(1-\beta)$ is the $(1-\beta)$-th quantile among $\{Z_s^*\}_{s=1}^S$.
2. Compute the vector $\tilde{\boldsymbol{\mu}}$ where, for $j = 1, \ldots P$:

$$\tilde{\mu}_j = \min\left\{\mu_j + K^{-1}(1-\beta), 0\right\}$$

3. Evaluate the $(1-\alpha+\beta)$ quantile of the test statistic $T(\cdot, \hat{\Sigma})$. We do this by simulation: draw vectors $X_s \sim N(\tilde{\mu}, \hat{\Sigma})$, for $s = 1, \ldots S$, and for each one, compute the test statistic $T_s \equiv T(X_s, \hat{\Sigma})$. Then the critical value is the $(1-\alpha+\beta)$-th quantile among $\{T_s\}_{s=1}^S$.

Table 7 shows the test results of checking the cyclic monotonicity conditions with our experimental data using the RSW approach. We see that the results are unchanged from those reported in Section 6.2 using the Andrews-Soares (2010) procedure.

Table 7: Testing for Cyclic Monotonicity in Experimental Data: Using Romano-Shaikh-Wolf (2014) Procedure

Data (all subjects)	**Test statistic**	$c^S_{1-\alpha+\beta}$
All cycles	68.194	39.436
Row cycles	68.194	31.594
Col cycles	0.000	5.119

Notes. All computations use $S = 5,000$. $\alpha = 0.05, \beta = 0.01$.

F. EXPERIMENT INSTRUCTIONS

The instructions in the experiment, given below, largely follow McKelvey et al. (2000).

This is an experiment in decision making, and you will be paid for your participation in cash. Different subjects may earn different amounts. What you earn depends partly on your decisions and partly on the decisions of others.

The entire experiment will take place through computer terminals, and all interaction between subjects will take place through the computers. It is important that you do not talk or in any way try to communicate with other subjects during the experiment. If you violate the rules, we may ask you to leave the experiment.

We will start with a brief instruction period. If you have any questions during the instruction period, raise your hand and your question will be answered so everyone can hear. If any difficulties arise after the experiment has begun, raise your hand, and an experimenter will come and assist you.

This experiment consists of several periods or matches and will take between 30 to 60 minutes. I will now describe what occurs in each match.

[Turn on the projector]

First, you will be randomly paired with another subject, and each of you will simultaneously be asked to make a choice.

Each subject in each pair will be asked to choose one of the three rows in the table which will appear on the computer screen, and which is also shown now on the screen at the front of the room. Your choices will be always displayed as rows of this table, while your partner's choices will be displayed as columns. It will be the other way round for your partner: for them, your choices will be displayed as columns, and their choices as rows.

You can choose the first, the second, or the third row. Neither you nor your partner will be informed of what choice the other has made until after all choices have been made.

After each subject has made his or her choice, payoffs for the match are determined based on the choices made. Payoffs to you are indicated by the red numbers in the table, while payoffs to your partner are indicated by the blue numbers. Each cell represents a pair of payoffs from your choice and the choice of your partner. The units are in francs, which will be exchanged to US dollars at the end of the experiment.

For example, if you choose 'A' and your partner chooses 'D', you receive a payoff of 10 francs, while your partner receives a payoff of 20 francs. If you choose 'A' and your partner chooses 'F', you receive a payoff of 30 francs, while your partner receives a payoff of 30 francs. If you choose 'C' and your partner chooses 'E', you receive a payoff of 10 francs, while your partner receives a payoff of 20 francs. And so on.

Once all choices have been made the resulting payoffs and choices are displayed, the history panel is updated and the match is completed.

[show the slide with a completed match]

This process will be repeated for several matches. The end of the experiment will be announced without warning. In every match, you will be randomly paired with a new subject. The identity of the person you are paired with will never be revealed to you. The payoffs and the labels may change every match.

After some matches, we will ask you to indicate what you think is the likelihood that your current partner has made a particular choice. This is what it looks like.

[show slide with belief elicitation]

Suppose you think that your partner has a 15% chance of choosing 'D' and a 60% chance of choosing 'E'. Indicate your opinion using the slider, and then press 'Confirm'. Once all subjects have indicated their opinions and confirmed them, the resulting payoffs and choices are displayed, the history panel is updated and the match is completed as usual.

Your final earnings for the experiment will be the sum of your payoffs from all matches. This amount in francs will be exchanged into U.S. dollars using the exchange rate of 90 cents for 100 francs. You will see your total payoff in dollars at the end of the experiment. You will also receive a show-up fee of $7. Are there any questions about the procedure?

[wait for response]

We will now start with four practice matches. Your payoffs from the practice matches are not counted in your total. In the first three matches you will be asked to choose one of the three rows of a table. In the fourth match you will be also asked to indicate your opinion about the likelihood of your partner's choices for each of three actions. Is everyone ready?

[wait for response]

Now please double click on the 'Client Multistage' icon on your desktop. The program will ask you to type in your name. Please type in the number of your computer station instead.

[wait for subjects to connect to server]

We will now start the practice matches. Do not hit any keys or click the mouse button until you are told to do so.

[start first practice match]

You see the experiment screen. In the middle of the screen is the table which you have previously seen up on the screen at the front of the room. At the top of the screen, you see your subject ID number, and your computer name. You also see the history panel which is currently empty.

We will now start the first practice match. Remember, do not hit any keys or click the mouse buttons until you are told to do so. You are all now paired with someone from this class and asked to choose one of the three rows. Exactly half of your see label 'A' at the left hand side of the top row, while the remaining half now see label 'D' at the same row.

Now, all of you please move the mouse so that it is pointing to the top row. You will see that the row is highlighted in red. Move the mouse to the bottom row and the highlighting goes along with the mouse. To choose a row you just click on it. Now please click once anywhere on the bottom row.

[Wait for subjects to move mouse to appropriate row]

After all subjects have confirmed their choices, the match is over. The outcome of this match, 'C'-'F', is now highlighted on everybody's screen. Also, note that the moves and payoffs of the match are recorded in the history panel. The outcomes of all of your previous matches will be recorded there throughout the experiment so that you can refer back to previous outcomes whenever you like. The payoff to the subject who chose 'C' for this match is 20, and the payoff to the subject who chose F is '10'.

You are not being paid for the practice session, but if this were the actual experiment, then the payoff you see on the screen would be money (in francs) you have earned from the first match. The total you earn over all real matches, in addition to the show-up fee, is what you will be paid for your participation in the experiment.

We will now proceed to the second practice match.

[Start second match]

For the second match, you have been randomly paired with a different subject. You are not paired with the same person you were paired with in the first match. The rules for the second match are exactly like for the first. Please make your choices.

[Wait for subjects]

We will now proceed to the third practice match. The rules for the third match are exactly like the first. Please make your choices.

[Start third match]

We will now proceed to the fourth practice match. The rules for the fourth match are exactly like the first. Please make your choices.

[Wait for subjects]

Now that you have made your choice, you see that a slider appears asking you to indicate the relative likelihood of your partner choosing each of the available actions. There is also a confirmation button. Please indicate your opinion by adjusting the thumbs and then press 'Confirm'.

[wait for subjects] This is the end of the practice match. Are there any questions? [wait for response]

Now let's start the actual experiment. If there are any problems from this point on, raise your hand and an experimenter will come and assist you. Please pull up the dividers between your cubicles.

[start the actual session]

The experiment is now completed. Thank you all very much for participating in this experiment. Please record your total payoff from the matches in U.S. dollars at the experiment record sheet. Please add your show-up fee and write down the total, rounded up to the nearest dollar. After you are done with this, please remain seated. You will be called by your computer name and paid in the office at the back of the room one at a time. Please bring all your things with you when you go to the back office. You can leave the experiment through the back door of the office.

Please refrain from discussing this experiment while you are waiting to receive payment so that privacy regarding individual choices and payoffs may be maintained.